11.2 Combinations

Recall: When the order of a selection or arrangement matters, we call it a permutation $\boxed{{}_nP_r = \frac{n!}{(n-r)!}}$

Today: With some arrangements or selections, order does not matter. These are known as combinations. $\boxed{{}_nC_r = \frac{n!}{(n-r)!\,r!}}$

ex) I am choosing a committee of president, VP and secretary, from 5 candidates.

Since order matters, this is a permutation.

$${}_nP_r = {}_5P_3 = \frac{5!}{(5-3)!} = \frac{5\times4\times3\times\cancel{2}\times\cancel{1}}{\cancel{2}\times\cancel{1}} = 60$$

If I decide to just choose a committee of 3 (from 5) with no assigned roles, it is a combination, since order doesn't matter.

$${}_5C_3 = \frac{5!}{((5-3)!\,3!)} = \frac{5\times4\times\cancel{3}\times\cancel{2}\times\cancel{1}}{(2\times1)\times(\cancel{3}\times\cancel{2}\times\cancel{1})} = 10$$

ex) In this class today there are 19 girls and 13 boys.

1. How many ways could I choose a group of 6? $_{32}C_6 = \frac{32!}{26!6!} = 906192$

2. What if the group must have 3B & 3G?

$$_{13}C_3 \times {}_{19}C_3 = 286 \times 969 = 277134$$

3. What if Emma has to be one of the girls?

$$_{13}C_3 \times {}_{18}C_2 \times {}_1C_1 = 43758$$

ex) Compare $_5C_4$ to $_5C_1$. Think of a situation for each. Make sense?

$$_5C_4 = \frac{5!}{(5-4)!4!} = \frac{5!}{1!4!} = 5 \qquad _5C_1 = \frac{5!}{(5-1)!1!} = \frac{5!}{4!1!} = 5$$

ex) Your exam has some choice. You must answer 2 out of 4 in part A, and 4 out of 5 in part B. How many combos?

$$ {}_4C_2 \times {}_5C_4 $$
$$ 6 \times 5 = 30 $$

What if I say you can do at least 4 questions in part B?

$$ {}_4C_2 \times ({}_5C_4 + {}_5C_5) $$
$$ 6 \times (5 + 1) $$
$$ 6 \times 6 = \underline{36} $$

ex) Simplify $\frac{{}_nC_5}{{}_{n-1}C_3} = \frac{\frac{n!}{(n-5)!5!}}{\frac{(n-1)!}{(n-1-3)!3!}}$

$n! = n \times \underbrace{n-1 \times n-2 \times n-3 \ldots}$
$n(n-1)!$

$(n-4)! = (n-4)(n-5)(n-6)\ldots$
$(n-4)(n-5)!$

$= \frac{n!}{(n-5)!5!} \times \frac{(n-4)!3!}{(n-1)!}$

$= \frac{n\cancel{(n-1)!}(n-4)\cancel{(n-5)!}\cancel{3!}}{\cancel{(n-5)!}5\times4\times\cancel{3!}\cancel{(n-1)!}}$

$= n(n-4)/20$

ex) Solve for n if $2({}_nC_2) = {}_{n+1}C_3$.

$2! = 2\times1$
$1! = 1$
$0! = 1$

$2.5! = ?$
$2.5\times1.5\times0.5\ldots?$

$(n+1)!$
$n+1 \times \underbrace{n \times n-1 \times n-2 \ldots}$

$\cancel{2}\left(\frac{n!}{(n-2)!\cancel{2!}}\right) = \frac{(n+1)!}{(n+1-3)!3!}$

$\frac{3!n!}{\cancel{(n-2)!}} = \frac{(n+1)!}{\cancel{(n-2)!}\cancel{3!}} \cancel{3!}$

$6n! = (n+1)!$
$6\cancel{n!} = (n+1)\cancel{n!}$
$6 = n+1$
$5 = n$

why is ${}_nP_r > {}_nC_r$?

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1-6, 8-11, 13, 14, 17-20, 23, C1, C3